



Notes – Hash Functions

- I. Introduction
 - a. Many things done with secret key cryptography can also be done with hash functions
 - b. These are one-way functions. $h(m) = d$, but there is no $h'(d)$ such that $h'(d) = m$
 - c. Example: Take n , look at the middle digit.
 - d. Given the computational power we have available, cannot find (m_1, m_2) such that $d_1 = d_2$
 - e. The size of d is fixed; the size of m is not.
 - f. With a hash of m bits, would need to search $2^{m/2}$ messages at random to find two with the same hash (on average)
 - g. Could potentially generate two such messages, but it's a lot of work to compute all those hash values
 - h. Convention: Use a 128-bit hash
 - i. A good hash function should look at every bit of the input, so each will be different
 - i. MD2 did this by splitting into bytes, since that's what the hardware supported
 - ii. MD4, MD5, SHA are all 32-bit for the same reason.
- II. Randomness
 - a. We want each bit of the output to be 1 half the time, so no information can be inferred
 - b. Sending two very similar inputs through should yield two completely different results.
 - c. Monte Carlo Simulations
 - i. Want to simulate random variables
 - ii. Usually picked in a sequence (though a lengthy one)
 - iii. Average = 0.5, uniform distribution
 - iv. Completely predictable, but it's not a secret
 - d. For picking keys:
 - i. Want numbers that can't be predicted
 - ii. Don't want to rely on time – generating two numbers simultaneously can yield two completely different numbers
 - iii. Usually use audio/video (or quantum computing) to get random data)
- III. Authentication
 - a. Generate some "random" number r_A ; send plaintext to the other party (B)
 - b. B computes the hash of $K_{AB} | r_A$.
 - c. Don't just want $h(K_{AB})$ because then somebody could observe the hash and use it themselves
 - d. A can then compute the same thing and compare the results
 - e. There's no need to decrypt anything, but it still proves B has the key
- IV. MIC
 - a. Message Authentication / Integrity Code
 - b. If you compute $MD(m)$ and send that to verify integrity because anybody could calculate $MD(m')$ and use that to spoof a message
 - c. Could use $MD(K_{AB} | m)$ but the way the hash works (from beginning to end, one chunk at a time), Trudy could add more text to the end without knowing the key (just continue the computation)
 - d. Use $h(k_{AB} | m | K_{AB})$
- V. One-Time Pad
 - a. $b_1 = MD(K_{AB} | IV)$
 - b. $b_2 = MD(K_{AB} | b_1)$ et cetera
 - c. Could even use $b_2 = MD(K_{AB} | c_1)$ for $c_1 = b_1 \oplus m_1$
 - d. So it's possible to do encryption using just a hash function
 - e. Why does this matter? Because of export controls, one cannot export encryption algorithms, but could use a hash function instead.
- VI. UNIX Passwords
 - a. Exactly eight bytes (if you enter more than that it's ignored)
 - b. Create a salt = 12 bit number (random)
 - c. Store salt | $MD(\text{salt} | \text{password})$

- d. Break the password into $m_1|m_2|\dots|m_n$
 - e. Encrypt 0 with m_1 as the key, encrypt that with m_2 as the key, et cetera
 - f. It is one-way, but it's not a very good hash function
- VII. MD2
- a. One of the earliest hash functions
 - b. Gives 128 bit results and does computations as bytes
 - c. Algorithm
 - i. Add bytes to the message until it's a multiple of 16 bytes
 - 1. Add $16 - (r \bmod 16)$ bytes of $(r \bmod 16)$ where r is the number of bytes in the original message
 - 2. Always add bytes, even if it's already a multiple of 16.
 - ii. Append an insecure MD2 checksum to the end
 - 1. Initialized to zero, 16 bytes long.
 - 2. For each byte in the message, $c_n = \pi(m_{nk} \oplus c_{n-1}) \oplus c_n$
 - 3. Where π is a permutation supposedly based on the actual number π
 - 4. Append the checksum to the end of the message
 - iii. Process the whole message
 - 1. Operate on 16-byte chunks
 - 2. Have a 48-byte quantity (digest | chunk | digest \oplus chunk)
 - 3. Do 18 passes over q , each time updating one byte
 - a. $c_{-1} = 0$ initially, then $c_{-1} = c_{47} + \text{pass number} \bmod 256$
 - b. $c_n = c_n \oplus \pi(c_{n-1})$
 - c. Concept: Update each byte based on the formulae above
 - d. After pass 17, the working digest becomes the digest for the beginning of the next chunk.
 - 4. A problem: Could end up doing two passes, but don't need to! Compute the checksum as you go, then feed it into the last chunk in the final pass