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MATH-054 (Puterbaugh)
1 February 2003

Notes – Chapter 2

Quantified Statements

Definitions

A <u>predicate</u> is a sentence that contains a finite number of variables, which becomes a statement when we replace the variables with values.
The <u>domain</u> of the predicate is the set of all possible replacements.

Definition

Let P(x) be a predicate, and let x have domain D. The <u>truth set</u> is the set of all things in D which make P(x) true. The truth set is written {x in d | P(x)}

Definition

Let P(x) be a predicate with domain D. A <u>universal statement</u> is of the form $\forall$ x, P(x) which is read "For all x in D, P of x" It is true when the truth set of P(x) = D. It is false if there is at least one element in D that makes P(x) false.

Example

P(x): |x| = x
(a) Domain: x 0. $\forall$x P(x) is true.
(b) Domain: all reals $\forall$x P(x) is false.

Definition

Let P(x) be a predicate with domain D. An <u>existential statement</u> is of the form $\exists$x in D, P(x) and is read "There exists an x in D, such that P of x." It is true as long as P(x) is true for one x in D. It is only false when P(x) is false for all x in D.

Example

All basketball players are taller than six feet.
Domain: NBA Players.
False: x = Mugsy Boggs and x is shorter than six feet.

Some basketball players are tall.
Domain: NBA Players
True: x = Shaq and x is taller than six feet.

Universal Conditional Statements

$\forall$x, P(x) $\Rightarrow$ Q(x)
For all integers n, if n is even, n^2 is even.

~($\forall$x, P(x)) $\Leftrightarrow$ $\exists$x, ~P(x)
~($\exists$x, P(x)) $\Leftrightarrow$ $\forall$x, ~P(x)

Examples

$\exists$ book b such that $\forall$ people p, p has read b.
"There is a book that everyone has read."
Negation: ($\forall$ books b)($\exists$ person p) $\ni$ p has not read b.

"There is a book that nobody has read"
($\exists$ book b) $\ni$ ($\forall$ people p) p has not read b.
Negation: ($\forall$ books b)($\exists$ person p) $\ni$ p has read b

Examples

"Everybody trusts somebody"
$(\forall$ people $p)(\exists$ person $q) \ni p$ trusts q

"Somebody trusts everybody"
$(\exists$ person $p) \ni (\forall$ people $q)$ p trusts q

If the square of an integer is even, then the integer is even.
$(\forall n \in \mathbb{Z})(n^2 \text{ even} \Rightarrow n \text{ even})$

Contrapositive: $(\forall n \in \mathbb{Z})(n \text{ not even} \Rightarrow n^2 \text{ not even})$

Converse: $(\forall n \in \mathbb{Z})(n \text{ even} \Rightarrow n^2 \text{ even})$

Inverse: $(\forall n \in \mathbb{Z})(n^2 \text{ not even} \Rightarrow n \text{ not even})$

Examples

$\forall x$ if $P(x)$ then $Q(x)$
$P(a)$ for some a
$\therefore Q(a)$
(Universal Modus Ponens)

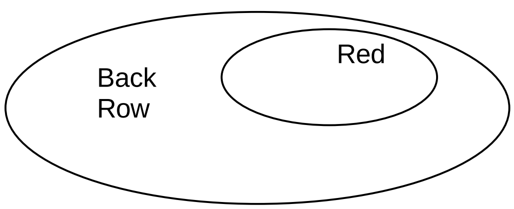
$\forall x$ if $P(x)$ then $Q(x)$
$\sim Q(a)$ for some a
$\therefore \sim P(a)$
(Universal Modus Tollens)

Theorem

If an integer n is odd then n^2 is odd.
$\forall x \in \mathbb{Z}$, n odd $\rightarrow n^2$ odd
16 is not odd $\therefore$ 4 is not odd

Diagrams

All redheaded people sit in the back row.
George is a redheaded person.
$\therefore$ George sits in the back row.



$p \rightarrow q$
$\sim q \rightarrow r$