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Notes – Chapter 1

Compound Statements

Definition

A statement (proposition) is a sentence that is either true or false, but not both.
→ "Tests are on Tuesdays." A statement – it's false.
→ "Today's weather is good." Not a statement.
→ "Stop." Not a statement.

Definition

If p is a statement variable, the negation of p, written ~p, is read "not p". Its truth value is the opposite of p.

p	~p
1	0
0	1

Definitions

Let p and q be statement variables.
The conjunction of p and q is written $p \wedge q$ and read "p and q." It is true only when both p and q are true. Otherwise it is false.
The disjunction of p and q is written $p \vee q$ and is read "p or q." It is false only when both p and q are false. Otherwise it is true.

p	q	$p \wedge q$	$p \vee q$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Example

$\sim(p \wedge q) \vee \sim r$

p	q	r	$p \wedge q$	$\sim(p \wedge q)$	~r	$\sim(p \wedge q) \vee \sim r$
1	1	1	1	0	0	0
1	1	0	1	0	1	1
1	0	1	0	1	0	1
1	0	0	0	1	1	1
0	1	1	0	1	0	1
0	1	0	0	1	1	1
0	0	1	0	1	0	1
0	0	0	0	1	1	1

Definition

Two statements are logically equivalent if they have the same truth values. If p and q are logically equivalent, we write $p \equiv q$

DeMorgan's Laws

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	~p	~q	$\sim p \wedge \sim q$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Definitions

A tautology is a statement that is always true.
A contradiction is a statement that is always false.

Example Quiz Question

Prove: If p and q are statement variables, then

1) $\sim(p \vee q) \equiv \sim p \wedge \sim q$
2) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Building the Proof

Ask, "What do we know? What does that mean? What does that mean?" Then ask "What do we want to conclude? What do we need for that? What do we need for that?" Build the two toward each other.

Answer to the example:
Proof: Let p and q be statement variables.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

Since $\sim(p \wedge q)$ has the same truth table as $\sim p \vee \sim q$ then $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

Distributive Laws

Let p, q, and r be statement variables. Then, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Proof: Let p, q, and r be statement variables. Then their truth tables are

p	q	r	$(q \vee r)$	$p \wedge (q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$(p \wedge q) \vee (p \wedge r)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Since the two have the same truth tables, they are equivalent.

Double Negation Law

Let p be a statement variable. $\sim(\sim p) \equiv p$

Definition

Let p and q be statement variables. The conditional of p by q is "If p, then q." or "p implies q." It is denoted by $p \rightarrow q$. It is false only when p is true and q is false.

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Theorem

Let p and q be statement variables. Then $p \to q \equiv {\sim}p \vee q$

Proof

Let p and q be statement variables.

p	q	$p \to q$	${\sim}p \vee q$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

Since $p \to q$ and ${\sim}p \vee q$ have the same truth tables, they are logically equivalent.

Theorem

Let p and q be statement variables. Then ${\sim}(p \to q) \equiv p \wedge {\sim}q$

Proof

${\sim}(p \to q) \equiv {\sim}({\sim}p \vee q) \equiv {\sim}({\sim}p) \wedge ({\sim}q) \equiv p \wedge {\sim}q$

Thus, ${\sim}(p \to q) \equiv p \wedge {\sim}q$

Example

Conditional: If it is snowing right now, then the temperature is below 32° Fahrenheit.

Negation: It is snowing and the temperature is greater than or equal to 32° Fahrenheit.

Definitions

Consider the conditional statement $p \to q$.

1) The <u>converse</u> of $p \to q$ is $q \to p$
2) The <u>inverse</u> of $p \to q$ is ${\sim}p \to {\sim}q$
3) The <u>contrapositive</u> of $p \to q$ is ${\sim}q \to {\sim}p$

Theorem

A conditional statement and its contrapositive are equivalent.

p	q	$p \to q$	${\sim}p$	${\sim}q$	${\sim}q \to {\sim}p$
1	1	1	0	0	1
1	0	0	0	1	0
0	1	1	1	0	1
0	0	1	1	1	1

The truth tables are the same, so the statements are equivalent.

Example

"If you vote for me, taxes will go down." Even if we know that to be true, we know nothing about the inverse: "If you don't vote for me, then taxes will not go down." It may be true or false.

Definition

Let p and q be statement variables. The <u>biconditional</u> is "p if and only if q." It is denoted by $p \leftrightarrow q$ and is true if p and q have the same truth values. It is abbreviated "p iff q."

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1