



Mutation and State

- I. Reference Cells
 - a. Most basic mutable entity
 - b. Essentially a memory location, similar to a pointer but completely transparent.
 - c. Example
 - i. $\text{let } x = \text{ref } 1;;$
 - ii. $!x$ (* dereference *)
 - iii. $x \Downarrow \{\text{mutable contents} = 1\}$
 - iv. $x := 2$ (* update, distinct from = *)
 - v. $!x \Downarrow 2$
 - d. No pointer arithmetic “or any nasty tricks like that”
 - e. Typing Rules
 - i. Creation: $\text{ref } e : \tau \text{ ref}$ iff $e : \tau$
 - ii. Dereferencing: $!e : \tau$ iff $e : \tau \text{ ref}$
 - iii. Assignments: $e := e' : \text{unit}$ iff $e : \tau \text{ ref}$ and $e' : \tau$
 - 1. Everything has a type. The choice of unit is arbitrary
 - 2. The purpose of () is as a placeholder. Anything that has a unit type generally signals the programmer that whatever is happening is side effectual.
 - f. Evaluation Rules
 - i. Creation: $\text{ref } e \Downarrow c$, where c is a reference cell, with v stored at c , where $e \Downarrow v$
 - ii. Dereferencing: $!e \Downarrow v$ iff $e \Downarrow c$ and v is currently stored at c
 - iii. Assignment
 - 1. $e := e' \Downarrow ()$ iff $e \Downarrow c$
 - 2. Side effect: Contents of c updated with v iff $e' \Downarrow v$
 - 3. In general, side effect is some effect of computation that's distinct from evaluation.
 - g. Example
 - i. $\text{let } x = \text{ref } 10$
 - ii. $x = 10$ NO! Type error ($\text{int} = \text{int ref}$)
 - iii. Aliasing
 - 1. $\text{let } y = \text{ref } x;;$
 - 2. $\text{let } z = \text{ref } x;;$
 - 3. $(!y) := 0$
 - 4. $!(!z) \Downarrow 0$
 - 5. $!x \Downarrow 0$
 - 6. “Mutation leads to aliasing, aliasing leads to pain.”
- II. Sequencing and Evaluation Order
 - a. Now the order in which statements are evaluated matters.
 - b. Sequencing: $e_1; e_2$ (first e_1 then e_2)
 - c. $e_1; e_2 : \tau$ iff $e_1 : \tau'$ and $e_2 : \tau$
 - d. Evaluation
 - i. $e_1; e_2 \Downarrow v$ iff $e_1 \Downarrow v'$ and $e_2 \Downarrow v$ (in that order) with implicit side-effects
 - ii. Value of e_1 is essentially thrown away!
 - iii. If e_1 does not have type unit, the compiler reports a warning.
 - e. Example
 - i. $\text{let } x = \text{ref } 0;;$
 - ii. $x := 1; x := 2; x := 3; !x;; \Downarrow 3$
 - iii. $\text{type } r = \{a : \text{unit}; b : \text{unit}\}$
 - iv. $\{a = (x := 1); b = (x := 2)\}$
 - v. $!x \Downarrow 1$
 - f. Example
 - i. $(x := 1; (\text{fun } x \rightarrow x))(x := 2; 1) \Downarrow 1$
 - ii. $!x \Downarrow 2$

- III. Mutable Records
- Slightly more complex form
 - `type mutrec = {mutable a : int; b : int};;`
 - `let mr = {a = 1; b = 2}`
 - `mr.a ↓ 1`
 - `mr.a <- 3`
 - `mr.a ↓ 3`
 - `mr.b <- 5` NO!
 - Reference cells are really just syntactic sugar for mutable records with one field.
 - `type 'a ref = {mutable contents : 'a}`
 - `ref e {contents = e}`
 - `!e ↓ e.contents`
 - `e := e' ↓ e.contents <- e'`
 - `# let a = ref 5;;`
 - `– val a : int ref = {contents = 5}`
- IV. The Values Restriction
- Example
 - `let x = ref (fun x -> x)`
 - `x : ('a -> 'a) ref`
 - `let f y = (!x)y : 'a -> 'a`
 - (Remember static typing!)
 - `x := (fun x -> x + 1)`
 - `f("uh-oh")` still *looks* valid but no longer makes sense!
 - This has been a major topic for years
 - Andrew Wright in 1995 proposed a solution known as "The Values Restriction"
 - Only values can have polymorphic types.
 - `let x = (fun x -> x : 'a -> 'a`
 - `let x = ref (fun x -> x)`
 - In standard ML this won't be allowed.
 - In OCaml it gets type `(_ 'a -> _ 'a)`
 - This is a placeholder that will be filled in once it gets used for the first time.
 - `let f y = (!x) y : _ 'a -> _ 'a)`
 - `f 1 ↓ 1`
 - Now the type of `f` is `(int -> int)`
 - The type of `x` is `(int -> int)`
 - `f("uh-oh")` NO! Not well typed!
 - This is a simple solution.
 - Many more complex ideas were suggested.
 - Ultimately a compiler was written to implement this solution and millions of lines of existing code were compiled.
 - Only a few places had problems and those problems were easily fixed.
 - The "restriction" didn't seem to restrict normal / correct use of the language, so it was adopted.
 - Eta-Conversion
 - Example
 - `let duplicate = map (fun x -> (x, x))`
 - `map : ('a -> 'b) -> 'a list -> 'b list`
 - `duplicate : _ 'a list -> _ 'a * _ 'a list`
 - It's no longer polymorphic!
 - The way around that problem is to wrap it in a function.
 - `let dupliate = (fun x -> map (fun x -> (x, x)))`
 - This is equivalent, but it's wrapped in a lambda abstraction so now it's a value (not an application) and the values restriction doesn't apply.

- V. Why Have Mutation?
- a. Purists say “no reason!”
 - b. `let rec fibnum x = match x with 0 -> 1 | 1 -> 1 | x -> fibnum(x - 1) + fibnum(x - 2)`
 - c. This is a naïve implementation – it’s extremely exponential.
 - d. Memoization: Uses mutation / mutable reference cells to store and lookup values previously computed. Now the function becomes linear.

ERROR: undefinedfilename
OFFENDING COMMAND:

STACK: