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Inventory Control

I. Introduction
   a. Inventory represents an investment
   b. it's listed as an asset on the balance sheet, but it's an undesirable asset
   c. There are some benefits to carrying inventory but there are many negatives
   d. Properly managed inventory yields benefits that outweigh costs
   e. Purpose of Inventory
      i. Buffers / decouples production from demand
      ii. Categorize inventory by its function
         1. Lot-Size Inventory: Economies of scale (get price reduction for large quantities purchased, so purchase more than you immediately need)
         2. Safety Stocks: Uncertainty (avoid shortages / stock-outs)
         3. Anticipation Inventories: Smoothing / speculation (if you suspect rising prices, buy more now)
         4. Pipeline Inventories: transportation / logistics
   f. Fundamental Questions
      i. When to order more of item X?
         1. Before you run out
         2. Make more or buy more
      ii. How much to order
   g. Considerations
      i. Characteristics of Demand
         1. Constant or variable?
         2. Known demand or random (forecasted? Under your control?)
      ii. Decision Costs
         1. Item Cost (per-unit)
         2. Setup / ordering cost
         3. So you have a variable and fixed cost. Shocking.
         4. Carrying / Holding cost: What does it cost to actually hold the inventory? May be expressed as a percentage of the unit cost
         5. Shortage / stock-out cost: Penalty for not having a needed item
            a. Profit lost from not having the item
            b. Where you have a backorder capability this is handled differently: Often send the item immediately, incur an extra shipping charge (include that)
            c. For internal ordering it's the cost of shutting down the production line or whatever other consequences rise from not having the item

II. Basic Economic Order Quantity (EOQ)
   a. Minimize Relevant Costs
   b. Find optimal ("economic") order quantity
   c. Average Annual Cost = Ordering + Purchase + Inventory Holding
   d. $G(Q) = k * (\lambda / Q) \quad + \quad c\lambda \quad + h(Q / 2)$
      i. k is the cost of placing the order
      ii. λ is the annual demand
      iii. Q is the order quantity
      iv. c is the unit cost
      v. h is the holding cost
      vi. $k(\lambda/Q)$ is the ordering cost
      vii. $c\lambda$ is the purchase cost
      viii. h(q/2) is the inventory holding cost (where Q/2 is average inventory)
   e. We don't actually care about $c\lambda$ since it's the same no matter what we do.
   f. An interesting fact:
      i. Ordering Cost = Holding Cost at Q* (the economic order quantity)

ii. $G(Q^*) = G^* =$ minimum cost
iii. $G^* = k\lambda/Q^* + hQ^*/2$
iv. $Q^* = \sqrt{(2k\lambda/h)}$
v. "And then a miracle happens" (as Far Side says) – do a bunch of algebra to get:
vi. $G^* = \sqrt{(k\lambda h/2)} + \sqrt{(k\lambda h/2)} = 2\sqrt{(k\lambda h/2)} = \sqrt{(2k\lambda h)}$

g. Cost Error: $G(Q) / G^* = \ldots = 0.5 (Q^*/Q + Q/Q^*)$
h. Suppose you choose $2Q^*$ as your order quantity
   i. The price will change based on this handy-dandy formula
   ii. For $Q = 2Q^*$, the "cost error" is $\frac{1}{2}(\frac{1}{2} + {}^{2}/_{1}) = {}^{5}/_{4} = 1.25$
   iii. So an order twice as large costs 25% more.
i. Likewise for $Q = 0.5Q^*$, it's 1.25 again

III. Finite Production Rate
a. When you're producing units, they don't all arrive at once – it takes time to produce them.
b. $G(Q) = k(\lambda/Q) + h(H/2)$ where H is the maximum inventory
c. This is just like before, but now we need to calculate the maximum inventory
d. Math
   i. p = production rate / year, λ = demand rate / year
   ii. T_1 = Time required for a production run
   iii. $T_1 = Q / p$
   iv. $H = (p = \lambda)T_1$ → Net build-up during the production run)
   v. $H = (p - \lambda)(Q / p) = ((p - \lambda) / p)Q = (1 - \lambda/p)Q$
   vi. So $G(Q) = k(\lambda/Q) + h(1 - \lambda/p)(Q/2)$
   vii. So let's just call $h' = h(1 - \lambda/p)$ and use h' in the same old formula from before

IV. Discounts on Orders
a. All-Units Discount
   i. Prices:
   1 – 99 sell at $500
   100 – 199 @ $490
   200 and up @ $475
   ii. So $C(Q) = \{ 500Q$ for $Q < 100$, $490Q$ for $100 \leq Q < 200$, $475Q$ for $Q \geq 200 \}$
   iii. Calculate the EOQ for each price bracket
   iv. Note that the EOQ for some price brackets isn't actually realizable, since you'd have to buy more units.
b. Incremental Discount Schedule
   i. Prices
   Units 1 – 100 sell at $500 each
   Units 101 – 200 @ $480 each
   Units 201 and up @ $460 each
   ii. So C(Q) = {
   $500Q$ for $Q \leq 100$
   $(500)(100) + 480(Q – 100)$ for $100 < Q \leq 200$
   $(500)(100) + (480)(100) + 460(Q – 200)$ for $Q > 200$
   }
c. Still have $G(Q) = k\lambda/Q + hQ/2 + c\lambda$
d. But now h and c depend on Q
e. Cost of Order percent: C(Q)/Q = {
   500 for $Q \leq 100$
   2000/Q for $100 < Q \leq 200$
   6000/Q for $Q > 200$
   }
f. Now $G(Q) = k\lambda/Q + (h_0 + h(C(Q)/Q))(Q/2) + (C(Q)/Q)\lambda$

V. Inventory Control with Uncertain Demand
a. News Vendor Problem
   i. Without knowing the demand in advance, how much should we order?

   ii. Given probable demand levels and weights
   iii. μ = SUM($x_i w_i$) = mean demand = E(x)
   iv. Generated using historical data plus some knowledge of the current market
   v. $\sigma = \sqrt{\sigma^2} = \sqrt{VAR(x)}$
   vi. $VAR(x) = E[(x-\mu)^2] = E(x^2 - 2x\mu + \mu^2) = E(x^2) - 2\mu E(x) + \mu^2 = E(x^2) - \mu^2$
b. Math
   i. Given an order quantity Q, increase it by one unit iff the expected cost of being able to sell it exceeds the expected cost of having it left-over
   ii. Expected Benefit: c_u P{is sold} = C_uP(D > Q}
   iii. Expected Cost: c_o P{not sold} = c_oP{D ≤ Q}
   iv. Solve when Expected Benefit = Expected Cost
   v. P{D ≤ Q} = c_u / (c_u+ c_o)   The cumulative probability
   vi. P{D ≤ Q} is the probability of meeting *all demand* if the order quantity is Q
   vii. Stocking Rule: Q* = min Q such that F(Q) ≥ c_u / (c_u+ c_o)
c. Three times this problem arises
   i. When orders need to be placed well in the advance of when the items will be sold
   ii. Spoilage or obsolescence occurs quickly
   iii. No inventory is carried over between periods
d. Still have to identify the minimum and maximum demand and assign a probability to everything in between. The normal distribution is a good choice
e. Need a "central tendency" (perhaps through exponential smoothing), then standard error (usually get that through most forecasting methodologies)
f. Can then find the cumulative probability via a standard normal table
g. Compared to Old Inventory Control: We thought of demand as ongoing, so inventory was replenished after decreasing gradually over time

VI. Uncertain Ordering
a. With traditional reordering we want to reorder such that the order arrives exactly when inventory runs out.
b. When the lead time is τ, the reorder point R = τ * λ.
   i. Want to have enough inventory left to last through τ
   ii. Rate of Demand * Time = Quantity
   iii. Make sure λ and τ are in the same time units!
c. Call $\lambda\tau = \mu$ → the average demand during the lead time
   i. Then suppose μ is distributed normally
   ii. We want a certain probability of meeting all demand during the lead time
   iii. If you fall short, won't meet any demand until a new order arrives
   iv. If you use R = μ, will have a 50% chance of falling short: an intolerably large risk
   v. So set R > μ somewhere, depending on what kind of shortage you can tolerate.
   vi. R - μ is the "safety stock"
      1. On average you expect to carry extra inventory at all times (never drop all the way to zero)
      2. It's just a buffer to protect against stock-outs
      3. Of course, you'll sometimes experience higher demand than expected and fall below the safety stock demand. Will sometimes have low demand and end up way above the safety.
      4. If demand is high enough you can still have a stock-out, it's just less probable now.
d. Determining R
   i. Done much like EOQ
   ii. Expected Annual Cost = h(average inventory) + kλ/Q + shortage cost
   iii. Average Safety Stock = R – μ
   iv. Shortage Cost

1. Stock-outs will only occur when demand during the lead time is greater than the reorder point.
2. Won't ever have a stock-out (by definition) before the reorder point
3. Only have to worry about a stock-out at the end of the lead time
4. Have λ/Q orders per year. For each there's an X% chance of a stock-out (which you yourself set in order to determine R initially)
5. So you have (X%)(λ/Q) stock-outs per year
6. The actual cost per unit short is p
7. But how many units will you actually be short?
8. We can call it n, but we don't really know n. Depends on your safety stock level
9. Shortage cost = $(\lambda/Q)p\ n(R)$ where n® is the number of units short on average as a function of R, which already incorporates the whole X% thing

v. $EAC = h((R - \mu) + Q/2) + k\lambda/Q + (\lambda/Q)p\ n(R)$
vi. $EAC = h(R - \mu) + H(Q / 2) + (k + p\ n(R)\ )\ \lambda/Q$
vii. Take a partial derivative and solve for Q.
viii. Get $Q^* = \sqrt{}(2\lambda(k + p\ n(R)\ ) / h)$
ix. $1 – F(R) = (Qh) / (p\lambda)$
   1. $1 – F(R)$ is the cumulative probability of a shortage
   2. Set R such that the probability of a shortage is $(Qh) / (p\lambda)$

x. So to find Q, need R. To find R, need Q. There's no miracle algebra to resolve that.. Have to take a "guess" (the old way) and then do iterations to improve it.